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A METHOD TO COMPARE SLURRY TRANSPORT MODELS

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Abstract: In dredging sand and gravel are often transported through pipelines mixed with water. In order to determine the pumping power required, the resistance has to be determined. The transport process is always an interaction between the pump and the resistance of the mixture transported. There are many models in literature to determine this resistance, expressed as the hydraulic gradient, but which model is suited for the case considered. The paper will give an overview of some of the most used models and show a method to compare these models and make a choice of the best suited model.

KEY WORDS: slurry transport, hydraulic gradient, heterogeneous, homogeneous, dredging.

1. INTRODUCTION

The theoretical transition velocity between the heterogeneous regime and the homogeneous regime (the Equivalent Liquid Model or ELM is assumed for this) gives a good indication of the excess pressure losses due to the solids. For normal dredging applications with large diameter pipes and rather high line speeds, this transition velocity will be at a line speed higher than the Limit Deposit Velocity and will often be near the operating range of the dredging operations. The excess head losses are in some way proportional to this transition velocity. For a $\mathbf{D_p=0.1\ m}$ diameter pipe most models match pretty well, due to the fact that most experiments are carried out in small diameter pipes and the models are fitted to the experiments. Although the different models may have a different approach, the resulting equations go through the same cloud of data points. Since there are numerous fit lines of numerous researchers it is impossible to cover them all, so a choice is made to compare Durand & Condolios (1952), Newitt et al. (1955), Fuhrboter (1961), Jufin & Lopatin (1966), Zandi & Govatos (1967), Turian & Yuan (1977), the SRC (Saskatchewan Research Counsil) model and Wilson et al. (1992) with the DHLLDV (Delft Head Loss & Limit Deposit Velocity) framework as developed by Miedema & Ramsdell (2013).

NOMENCLATURE

$\mathbf{C_D}$	Particle drag coefficient	-
$\mathbf{C_L}$	Lift coefficient	-
$\mathbf{C_{vt}}$	Delivered (transport) volumetric concentration	-
$\mathbf{C_{vs}}$	Spatial volumetric concentration	-
$\mathbf{C_{vb}}$	Spatial volumetric concentration bed (1-n)	-
$\mathbf{C_x}$	Durand & Condolios reversed particle Froude number	-

$\mathbf{d}$	Particle diameter	**m**
$\mathbf{d_m}$	Mean particle diameter Fuhrboter	**m**
$\mathbf{d_{50}}$	Particle diameter with 50% passing	**m**
$\mathbf{D_p}$	Pipe diameter	**m**
$\mathbf{f_l}$	Fanning friction factor liquid	**-**
$\mathbf{f_m}$	Fanning friction factor mixture	**-**
$\mathbf{F_L}$	Lift force on particle	**N**
$\mathbf{F_W}$	Submerged weight of particle	**N**
$\mathbf{g}$	Gravitational constant 9.81 m/s^2	$\mathbf{m/s^2}$
$\mathbf{i_l}$	Pure liquid hydraulic gradient	**m/m**
$\mathbf{i_m}$	Mixture hydraulic gradient	**m/m**
$\mathbf{K}$	Durand & Condolios constant	**-**
$\mathbf{K_1}$	Newitt constant	**-**
$\mathbf{L_R}$	Lift force to submerged weight ratio	**-**
$\mathbf{n}$	Porosity	**-**
$\mathbf{\Delta p_l}$	Pressure loss liquid	**kPa**
$\mathbf{\Delta p_m}$	Pressure loss mixture	**kPa**
$\mathbf{R_{sd}}$	Relative submerged density	**-**
$\mathbf{v_{ls}}$	Line speed	**m/s**
$\mathbf{v_{ls,hh}}$	Intersection velocity heterogeneous-homogeneous regimes	**m/s**
$\mathbf{v_t}$	Terminal settling velocity particle	**m/s**
$\boldsymbol{\beta}$	Richardson & Zaki hindered settling power	**-**
$\boldsymbol{\lambda_l}$	Darcy Weisbach friction factor liquid	**-**
$\boldsymbol{\lambda_m}$	Darcy Weisbach friction factor mixture	**-**
$\boldsymbol{\mu_{sf}}$	Sliding friction factor	**-**
$\boldsymbol{\rho_l}$	Density liquid	$\mathbf{ton/m^3}$
$\boldsymbol{\rho_s}$	Density solid	$\mathbf{ton/m^3}$
$\boldsymbol{\nu_l}$	Kinematic viscosity liquid	$\mathbf{m^2/s}$
$\boldsymbol{\Phi}$	Durand & Condolios parameter	**-**
$\boldsymbol{\psi}$	Durand & Condolios parameter	**-**
$\boldsymbol{\zeta}$	Fit function Fuhrboter	**-**

2. THE MODELS CONSIDERED

The theoretical transition velocity can be determined by making the relative excess pressure contributions of the heterogeneous regime and the homogeneous regime equal. This is possible if transition effects are omitted and the basic equations are applied.

For the DHLLDV framework, this equation implies that the transition velocity depends linear on the pipe diameter and reversely on the viscous friction coefficient $\boldsymbol{\lambda_l}$. Since the viscous friction coefficient $\boldsymbol{\lambda_l}$ depends reversely on the pipe diameter $\mathbf{D_p}$ with a power of about **0.2**, the transition velocity will depend on the pipe diameter with a power of about **(1.2/4) =0.35.** The equation derived is implicit and has to be solved iteratively. This gives for the transition velocity between the heterogeneous regime and the homogeneous regime for the DHLLDV framework:

$$v_{ls,hh}^{4}=\frac{2\cdot g\cdot D_{p}}{\lambda_{l}}\cdot\left(\begin{array}{l} v_{t}\cdot\left(1-\dfrac{C_{vs}}{0.175\cdot(1+\beta)}\right)^{\beta}\cdot v_{ls,hh} \\ +\dfrac{7.5^{2}}{\lambda_{l}}\cdot\left(\dfrac{v_{t}}{\sqrt{g\cdot d}}\right)^{8/3}\cdot(\nu_{l}\cdot g)^{2/3}\end{array}\right) \tag{1}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Durand & Condolios (1952) and later Gibert (1960) is (The Wasp et al. (1977) related models also follow this equation):

$$v_{ls,hh}^{3}=\frac{K}{R_{sd}}\cdot\left(\frac{g\cdot D_{p}\cdot R_{sd}}{\sqrt{C_{x}}}\right)^{3/2}\quad \text{with:}\quad K=85,\ \sqrt{C_{x}}=\frac{\sqrt{g\cdot d}}{v_{t}} \tag{2}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Newitt et al. (1955) is:

$$v_{ls,hh}^{3}=K_{1}\cdot\left(g\cdot D_{p}\right)\cdot v_{t}\quad \text{with:}\quad K_{1}=1100 \tag{3}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Fuhrboter (1961) is, using an approximation for the S_k value:

$$v_{ls,hh}^{3}=\frac{2\cdot g\cdot D_{p}}{\lambda_{l}\cdot R_{sd}}\cdot\left(43.5\cdot\sqrt{C_{x}}^{-1}\cdot R_{sd}\cdot\zeta(d_{m})\cdot(\nu_{l}\cdot g)^{1/3}\right)=\frac{2\cdot g\cdot D_{p}}{\lambda_{l}\cdot R_{sd}}\cdot S_{k} \tag{4}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Jufin & Lopatin (1966) is, with added terms to make the dimensions correct:

$$v_{ls,hh}^{3}=2\cdot 82654\cdot\left(\frac{v_{t}}{\sqrt{g\cdot d}}\right)^{3/4}\cdot\left(\frac{g\cdot D_{p}}{R_{sd}}\right)^{1/2}\cdot(\nu_{l}\cdot g)^{2/3}\cdot\left(C_{vt}\right)^{-1/2} \tag{5}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Zandi & Govatos (1967) is:

$$v_{ls,hh}^{3.86}=280\cdot\left(\frac{g\cdot D_{p}\cdot R_{sd}}{\sqrt{C_{x}}}\right)^{1.93}\cdot\frac{1}{R_{sd}} \tag{6}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Turian & Yuan (1977), with saltating transport can be described by:

$$\mathbf{v_{ls,hh}} = \left(\mathbf{g} \cdot \mathbf{R_{sd}} \cdot \mathbf{D_p} \left(\begin{array}{l} \mathbf{107.1} \cdot \mathbf{C_{vt}^{1.018}} \cdot \left(\dfrac{\boldsymbol{\lambda_l}}{\mathbf{4}} \right)^{\mathbf{0.046}} \\ \cdot \mathbf{C_D^{*\,-0.4213}} \cdot \left(\mathbf{R_{sd}} \cdot \mathbf{C_{vt}} \right)^{-1} \end{array} \right)^{\mathbf{1/1.354}} \right)^{\mathbf{0.5}} \tag{7}$$

The transition velocity between the heterogeneous regime and the homogeneous regime of Turian & Yuan (1977), with heterogeneous transport can be described by:

$$\mathbf{v_{ls,hh}} = \left(\mathbf{g} \cdot \mathbf{R_{sd}} \cdot \mathbf{D_p} \left(\begin{array}{l} \mathbf{30.11} \cdot \mathbf{C_{vt}^{0.868}} \cdot \left(\dfrac{\boldsymbol{\lambda_l}}{\mathbf{4}} \right)^{\mathbf{0.200}} \\ \cdot \mathbf{C_D^{*\,-0.1677}} \cdot \left(\mathbf{R_{sd}} \cdot \mathbf{C_{vt}} \right)^{-1} \end{array} \right)^{\mathbf{1/0.6938}} \right)^{\mathbf{0.5}} \tag{8}$$

The simplified equation for heterogeneous transport of Wilson et al. (1992) for non-uniform PSD's gives:

$$\mathbf{v_{ls,hh}^{3}} = \mathbf{44.1} \cdot \frac{\boldsymbol{\mu_{sf}} \cdot \mathbf{g} \cdot \mathbf{D_p}}{\boldsymbol{\lambda_l}} \cdot \left(\mathbf{d_{50}} \right)^{\mathbf{0.35}} \tag{9}$$

The simplified equation for heterogeneous transport of Wilson et al. (1992) for uniform PSD's gives:

$$\mathbf{v_{ls,hh}^{3.7}} = \mathbf{44.1^{1.7}} \cdot \frac{\boldsymbol{\mu_{sf}} \cdot \mathbf{g} \cdot \mathbf{D_p}}{\boldsymbol{\lambda_l}} \cdot \left(\mathbf{d_{50}} \right)^{\mathbf{0.35 \cdot 1.7}} \tag{10}$$

It should be mentioned that this is based on the simplified model of Wilson et al. (1992) for heterogeneous transport. The full model may give slightly different results.

The SRC model (Shook & Roco, 1991) consists of two terms regarding the excess hydraulic gradient. A term for the contact load and a term for the suspended load. Here only the term for the contact load is considered, assuming the contact load results in a small bed and there is no buoyancy from the suspended load. The SRC model results in an implicit relation if only the contact load is considered, this gives:

$$\mathbf{v_{ls,hh}^{2}} \cdot \mathbf{e}^{\mathbf{0.0212} \cdot \frac{\mathbf{v_{ls,hh}}}{\mathbf{v_t}}} = \mu_{\mathbf{sf}} \cdot \frac{\mathbf{2} \cdot \mathbf{g} \cdot \mathbf{D_p}}{\lambda_{\mathbf{l}}} \tag{11}$$

3. EXAMPLES HETEROGENEOUS VERSUS HOMOGENEOUS

The comparison is based on models for saltating or heterogeneous transport. As mentioned before, the transition line speed from heterogeneous to homogeneous transport is a good indicator for the excess pressure losses. A higher transition line speed indicates higher excess pressure losses. For a pipe diameter of 0.1016 m (4 inch) and medium sized particles (0.1 mm to 2 mm) all models are close for high concentrations (around 30%), shown in Fig 2, which is used as a reference graph. This is caused by the fact that most experiments are carried out in small diameter pipes, resulting in a cloud of data points because of scatter. Many curves will fit through this cloud of data points. The Limit Deposit Velocity curve (according to DHLLDV) and the transition to the sliding flow regime ($\mathbf{d>0.015 \cdot D_p}$), see Wilson et al. (1992), are also shown.

The following graphs (Fig 1, Fig 2 and Fig 3) have a dimensionless vertical axis by dividing the transition line speed by the maximum transition line speed occurring in one of the 11 models. This maximum transition line speed is shown in the lower right corner of each graph. For normal dredging operations, particles diameters from 0.1 mm up to 10 mm are of interests. The different models are compared with the DHLLDV framework of Miedema & Ramsdell (2013). For very small pipe diameters (<0.1 m) the Newitt et al. (1955) model is representative, for medium pipe diameters (0.1-0.3 m) the Fuhrboter (1961) model and the Durand & Condolios (1952) model are representative and for large pipe diameters (>0.3 m) the Jufin & Lopatin (1966) model is representative, although this model tends to underestimate the pressure losses slightly for large pipe diameters and high concentrations. The Wilson et al. (1992) and the SRC models have developed over the years and are based on many experimental data and can thus be considered to be representative in all cases for medium sized particles. For the 0.0254 m (1 inch) pipe, in the range of medium sized particles most models are very close. The Durand & Condolios model however seems to underestimate the transition velocity, while the Jufin & Lopatin and the Turian & Yuan 2 models overestimate the transition velocity. The SRC and the DHLLDV models are very close up to a particle size of about 2 mm. For the 0.1016 m (4 inch) pipe, most models are close for medium sized particles for higher concentrations. The Turian & Yuan 2 and the Jufin & Lopatin models depend on the concentration. The SRC and DHLLDV models match closely up to a particle diameter of about 1 mm. The Wilson et al. models matches both models for particle diameters close to 1 mm.

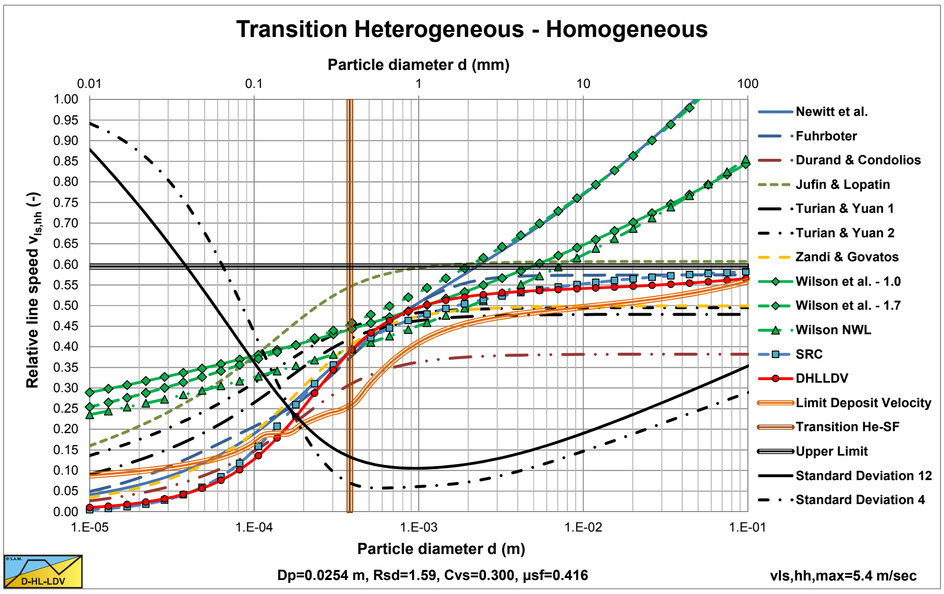

Fig 1 The transition line speed He-Ho for D_p=0.0254 m and C_v=0.30.

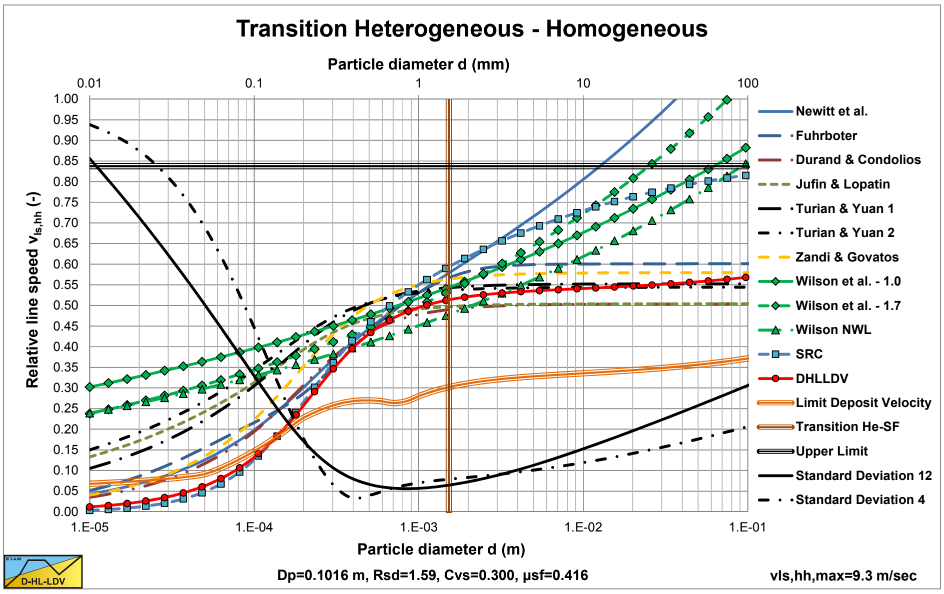

Fig 2 The transition line speed He-Ho for D_p=0.1016 m and C_v=0.30.

For the 0.762 m (30 inch) pipe, the models deviate strongly. Durand & Condolios, Zandi & Govatos and Turian & Yuan 1 and 2 give high transition velocities. Fuhrboter and Wilson et al. -1.0 medium transition velocities, while DHLLDV, SRC, Wilson et al. -1.7 and Jufin & Lopatin (medium concentrations) give low transition velocities.

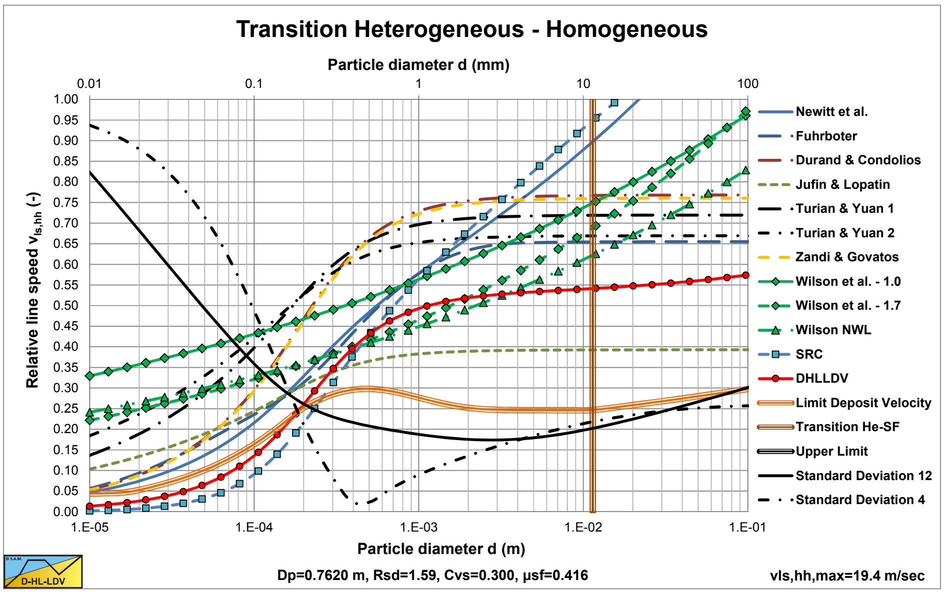

Fig 3 The transition line speed He-Ho for D_p=0.762 m and C_v=0.30.

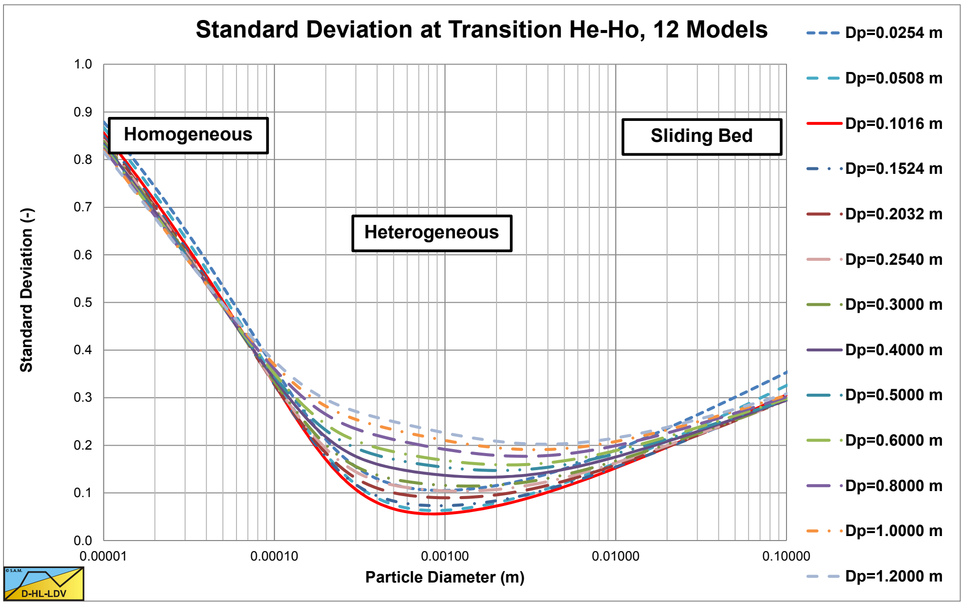

Figure 4 The standard deviation of 12 models.

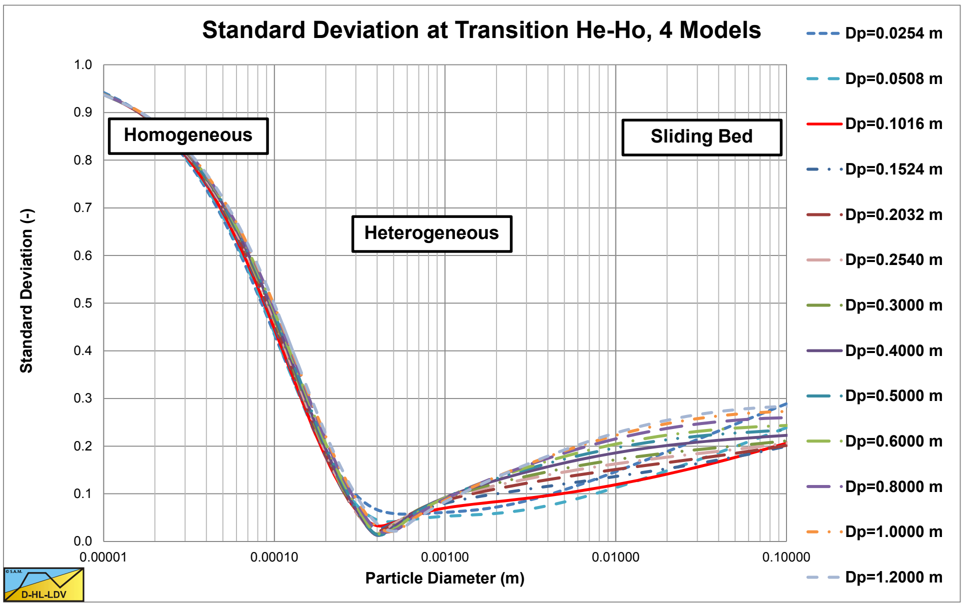

Figure 5 The standard deviation of 4 models.

Figure 4 shows the standard deviation of all 12 models. For pipes with a diameter close to 0.1016 m (4 inch) and particles with a diameter in the range of 0.3 mm to 3 mm the standard deviation is below 10% and the models give a comparable result in terms of the solids effect. For much smaller or larger pipes the standard deviation increases, which is also the case for very small or large particles.

Figure 5 shows the standard deviation of the DHLLDV framework, the SRC model and the Wilson et al. models with a power of 1.7 and the near wall lift method (NWL). In the range of particle diameters from 0.25 mm to 1-2 mm the standard deviation is less than 10% and does not really depend on the pipe diameter. Very small particles show a high standard deviation, but the heterogeneous models are not applicable there. At normal operational line speeds, which are much higher than the transition line speeds found, the transport will be according to the homogeneous flow regime (ELM related). Very large particles show an increasing standard deviation, resulting from the fact that here the sliding bed or sliding flow regimes are applicable and not the heterogeneous flow regime.

Although the physics behind the 4 models considered, the DHLLDV framework, the SRC model and the 2 Wilson et al. models, are different, the 4 models give about the same result for medium sands in terms of the solids effect, irrespective of the pipe diameter.

4. CONCLUSIONS & DISCUSSION

The transition velocity of the heterogeneous regime equation with the ELM equation seems to be a good indicator for comparing the different models. A requirement is of course that the heterogeneous component can be isolated. The hydraulic gradient or head losses can be easily determined at the transition velocity, since the ELM is valid in this intersection point. The solids effect of the hydraulic gradient or head losses is proportional to the transition velocity squared. Based on the amount of experimental data, the SRC model and the Wilson et al. models seem to be the most reliable models from literature. The DHLLDV framework is very close to these models for medium sized particles, although DHLLDV is based on kinetic energy losses, while both SRC and Wilson et al. are based on a diminishing bed.

Models are based on the terminal settling velocity or models are based on the particle Froude number. The difference between the two groups is, that the terminal settling velocity continuously increases with the particle diameter, while the particle Froude number and the particle drag coefficient have a constant asymptotic value for large particle diameters. So models from the first group tend to overestimate the transition line speed for large particles. This is however not surprising, since both Newitt et al. (1955), SRC and Wilson et al. (1992) use a 2LM or sliding bed model for this case, so the large particle part of the curves is not relevant.

The small particle part of the graphs is also not relevant, since the transport regime will be homogeneous at normal operational line speeds, which are much higher than the transition line speeds found.

In general one can say that the transition line speed is proportional to the pipe diameter with a power of 0.3 to 0.4, based on the Wilson et al. (1992) models, the SRC model and the DHLLDV framework.

In general one can say that the transition line speed depends weakly on the spatial concentration.

In general one can say that there is no or hardly no influence of the sliding friction coefficient on the transition velocity of the heterogeneous and the homogeneous regimes.

In general one can say that there is a weak or no direct dependency of the transition velocity of the heterogeneous and the homogeneous regimes on the relative submerged density. Some models give a weak indirect dependency on the particle Froude number.

In general one can say that the more negative the power of the line speed in the heterogeneous hydraulic gradient equation, the smaller the transition velocity between the regimes. One has to take this into account interpreting the graphs.

The DHLLDV framework is a very good alternative to the well-known models from literature for particles in the range of 0.1 mm up to a few mm.

More graphs and other information can be found on www.dhlldv.com.

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